Trigonometric Analysis of the Mechanical Axis Deviation Induced by Telescopic Intramedullary Femoral Lengthening Nails

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Femoral lengthening with intramedullary nails can create alterations in the mechanical axis of the limb. This is based on the relationship of the anatomic femur axis to the mechanical femur axis, which is typically 5–9 degrees valgus. We developed trigonometric formulas to calculate the predicted change, using the lengths of the tibia, femur, and whole limb; the amount of lengthening; and the angle between the anatomic and the mechanical axis of the femur. We recognized three patterns depending on whether the overall limb mechanical axis is lateral (valgus), medial (varus), or straight through the center of the knee. The varus and valgus patterns lead to similar formulas. When the mechanical axis goes directly through the center of the knee joint, the formula simplifies. These formulas could be incorporated into digital radiographic programs to predict the change in mechanical axis deviation that will develop from lengthening along the anatomic femur axis with an intramedullary lengthening nail.

Keywords: mechanical axis deviation, alignment, limb lengthening, intramedullary nail, femur

Femoral lengthening has been performed since the 1920s (Putti, 1921) by means of various types of external fixators that must stay in place 4–6 months or longer (Ilizarov, 1988; Paley, 1988; Vilarrubias et al., 1990). In an effort to make the lengthening experience more comfortable, internal lengthening devices for the femur and tibia have been developed (Baumgart et al., 1997; Cole et al., 2001; Guichet & Casar, 1997). These are self-lengthening telescopic intramedullary rods, which do not need an external apparatus to make them extend.

In normal femora, the mechanical and anatomic femoral axes diverge by approximately 5–9°. This angle is known as anatomic-mechanical angle (AMA) (Paley, 2002). This causes lateralization of the overall limb alignment while lengthening the femur along its anatomic axis. This phenomenon does not occur in the tibia because the tibial anatomic and mechanical axes are parallel (Paley, 2002). The purpose of this study was to develop a trigonometric formula to predict the change in the overall limb mechanical axis that occurs from femoral lengthening with telescopic nails.

Methods

We modeled three possible scenarios: no mechanical axis deviation (MAD), lateral MAD (valgus), or medial MAD (varus) (Figure 1). To generate formulae to calculate the predicted axis shift caused by lengthening along the anatomic femur axis, certain parameters have to

Figure 1 — Three clinical scenarios for prelengthening limb alignment. Left: Valgus configuration showing lateralization of the mechanical axis. Center: Normal alignment with the mechanical axis passing through the center of the knee. Right: Varus configuration showing medialization of the mechanical axis. (Reprinted with permission, Sinai Hospital of Baltimore.)
be defined. These include the lengths of the tibia, the femur, and the total limb; the AMA; and the amount of lengthening. Using these five parameters for every AMA and every amount of lengthening, the predicted shift of the mechanical axis can be calculated independent of the degree of valgus or varus deformity.

Results

Scenario 1: Normal Alignment

Figure 2 — Schematic of the normal lower limb with no initial mechanical axis deviation, demonstrating the required measurements. The length of the femoral neck has been deliberately exaggerated in order to make the geometric relationships more clear. (Reprinted with permission, Sinai Hospital of Baltimore.)
The simplest scenario is the case of no initial mechanical axis deviation (Figure 2). The calculation in this case is explained with the theorem of similar triangles and results in Equation 1. Figure 2 is an approximation, as two right angles in one triangle are impossible. However, because the acute angle $\angle BAG$ is extremely small (typically less than 1°), it is acceptable to draw the line $BG$ with a right angle on both sides.

$$\frac{e'}{\Delta e} = \frac{a + d + f}{a}$$

$$\Delta e = \frac{a \cdot e'}{a + d + f}$$

with $\sin \alpha = \frac{e'}{\Delta b}$ and $\cos \alpha = \frac{f}{\Delta b}$

$$\Delta e = \frac{a \cdot \Delta b \cdot \sin \alpha}{a + d + \Delta b \cdot \cos \alpha} \quad (1)$$

### Scenario 2: Valgus Configuration

The second scenario is a preexisting valgus deformity (Figure 3). The aim is to define three lines or angles in the triangle $AGB$ (Figure 3), thereby defining the length of $\Delta e$.

Step 1:

$$a^2 = d^2 + g^2 - 2dg \cdot \cos \varepsilon$$

$$\cos \varepsilon = \frac{d^2 + g^2 - a^2}{2dg}$$

Step 2:

$$d^2 = a^2 + g^2 - 2ag \cdot \cos \omega$$

$$\cos \omega = \frac{a^2 + g^2 - d^2}{2ag} = \frac{g_1}{a}$$

$$\Rightarrow g_1 = a \cdot \cos \omega = \frac{a^2 + g^2 - d^2}{2g} \quad (2)$$

The first step is calculated with the law of cosines for nonorthogonal triangles: $a^2 = d^2 + g^2 - 2dg \cdot \cos \varepsilon$. The second step also follows the law of cosine for nonorthogonal triangles: $d^2 = a^2 + g^2 - 2ag \cdot \cos \omega$ and $\cos \omega = g_1/a$ arise from the nonorthogonal triangle $ADB$. The result is Equation 2.

Step 3:

$$\varphi = \beta + \gamma - \varepsilon$$

$$= \left(180 - \alpha - \beta\right) + \beta - \varepsilon$$

$$= 180 - \alpha - \varepsilon$$

Step 4:

$$\frac{\sin \theta}{\sin \varphi} = \frac{\Delta b}{x}$$
Figure 3 — Schematic of the lower limb in severe valgus, demonstrating the required measurements. The length of the femoral neck has been deliberately exaggerated in order to make the geometric relationships more clear. (Reprinted with permission, Sinai Hospital of Baltimore.)
with \[ x^2 = \Delta b^2 + g^2 - 2 \Delta b \, g \cos \varphi \]

\[
\sin \vartheta = \frac{\Delta b \sin \varphi}{\sqrt{\Delta b^2 + g^2 - 2 \Delta b \, g \cos \varphi}}
\]

Step 3 arises from the sum of angles from Figure 3 (\( \varphi = \beta \)).

Step 4 is composed of the law of sines, \((\Delta b/\sin \vartheta = x/\sin \varphi)\), and the law of cosines, \((a^2 = d^2 + g^2 - 2dg \cdot \cos e)\), to solve the auxiliary number “\(x\)” leading to Equation 3.

Step 5:

\[
\tan \vartheta = \frac{\Delta e}{g_1}
\]

\[
\Delta e = g_1 \tan \vartheta
\]

Step 5 is calculated according to the law of tangent leading to Equation 4.

Equations 2, 3, and 4 combined lead to the final Equation 5.

\[
\Delta e = \frac{a^2 + g^2 - d^2}{2g} \cdot \tan \left[ \arcsin \left( \frac{\Delta b \sin \left( 180 - \alpha - \arccos \frac{d^2 + g^2 - a^2}{2dg} \right) }{\sqrt{\Delta b^2 + g^2 - \cos \left( 180 - \alpha - \arccos \frac{d^2 + g^2 - a^2}{2dg} \right) }} \right) \right]
\]

Scenario 3: Varus Configuration

The third scenario is a preexisting varus deformity (Figure 4).

\[
\cos \varphi = \frac{d^2 + g^2 - a^2}{2dg}
\]

\[
\cos \omega = \frac{a^2 + g^2 - d^2}{2ag} = \frac{g_1}{a}
\]

\[
\Rightarrow \ g_1 = a \cdot \cos \omega = \frac{a^2 + g^2 - d^2}{2g}
\]

\[
\varphi = \beta + \gamma + \varepsilon
\]

\[
= 180 - \alpha + \varepsilon
\]

\[
\sin \vartheta = \frac{\Delta b}{x} \sin \varphi
\]

with \[ x^3 = \Delta b^3 + g^3 - 2 \Delta b \, g \cos \varphi \]

\[
\sin \vartheta = \frac{\Delta b \sin \varphi}{\sqrt{\Delta b^2 + g^2 - 2 \Delta b \, g \cos \varphi}}
\]

\[
\tan \vartheta = \frac{\Delta e}{g_1}
\]

\[
\Delta e = g_1 \tan \vartheta
\]

\[
\Delta e = \frac{a^2 + g^2 - d^2}{2g} \cdot \tan \left[ \arcsin \left( \frac{\Delta b \sin \left( 180 - \alpha + \arccos \frac{d^2 + g^2 - a^2}{2dg} \right) }{\sqrt{\Delta b^2 + g^2 - 2 \Delta b \, g \cos \left( 180 - \alpha + \arccos \frac{d^2 + g^2 - a^2}{2dg} \right) }} \right) \right]
\]
Figure 4 — Schematic of lower limb with varus deformity, demonstrating the required measurements. The length of the femoral neck has been deliberately exaggerated in order to make the geometric relationships more clear. (Reprinted with permission, Sinai Hospital of Baltimore.)
The calculations for varus deformity (Figure 4) are very similar to the second scenario with valgus deformity (Figure 3) except that the triangle \( AGB \) in Scenario 2 is reflected over the \( y \)-axis to create triangle \( AGB \) for Scenario 3. In addition, the angle \( \omega \) is added in Scenario 3, whereas in Scenario 2 it is subtracted. Unlike Equation 5, in which every term involving \( \arccos \) is subtracted, the \( \arccos \) term is added throughout Equation 6.

**Discussion**

The three geometric formulae are mathematically complex, but they are able to precisely predict the theoretical mechanical axis deviation that occurs after lengthening along the femoral anatomic axis. The MAD that is calculated using the formula is only as accurate as the measured parameters that are used in the formula. The most important parameter is the AMA. The AMA is normally a very small angle (5–9°), which makes it somewhat difficult to measure precisely in a clinical setting. An error of a few degrees will cause a significant error in the predicted MAD change. If the AMA is measured precisely but the nail is inserted so that it deviates even 1° from the anatomic axis, it will also cause inaccuracy of the predicted MAD change. The potential for every parameter to be measured imprecisely may reduce the validity and the clinical impact of the method.

Other factors may reduce the accuracy of the theoretical prediction. After an osteotomy, the femur may have a tendency to bend into varus because of the patient’s body weight and the adductor muscle force vector. Other important factors include the level of the osteotomy, the length of the nail, the width of the nail, and the width of the intramedullary canal. For example, a short, narrow nail within a wide intramedullary canal has more space to move and might create a medial shift of the MAD. This would lessen the lateral shift of the mechanical axis caused by lengthening (Figure 5).

Digital radiographic systems are becoming more common in orthopedic practice. These systems include tools that can accurately measure the five parameters: lengths of the tibia, femur, and total limb; AMA; and the amount of lengthening. The formulae presented in this article could be included in a digital radiographic system so that a clinician could quickly and easily predict the MAD after lengthening a given amount along the anatomic axis.

**Acknowledgment**

We would like to express our gratitude to Joe Michalski and Alvin Lee for support in artwork and Amanda Chase for editorial support.

**References**


